Closing Tues: 14.1
Closing Thur: 14.3(1), 14.3(2)
Closing *next* Tues: 14.4, 14.7

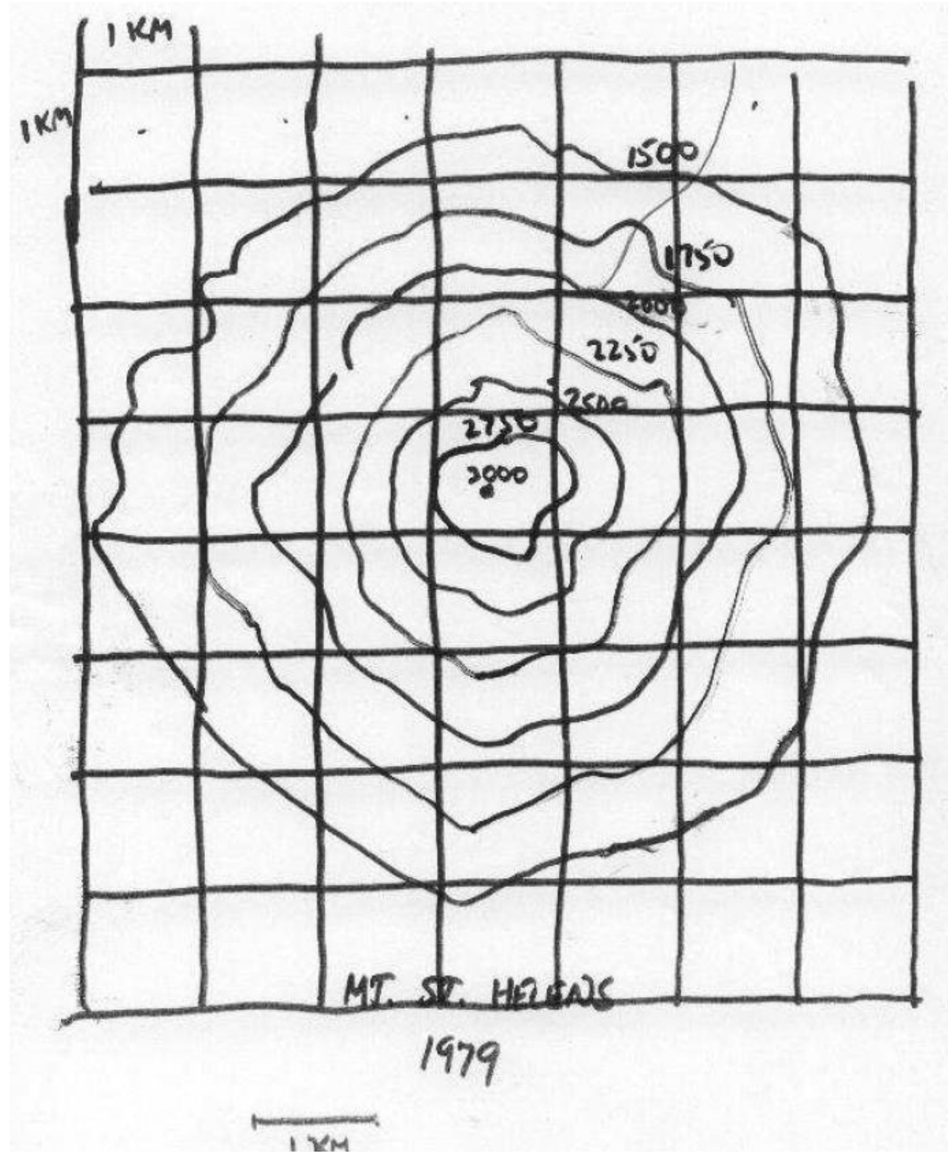
14.1/14.3 Visualizing Surfaces and Partial Derivatives

The basic tool for visualizing surfaces is **traces**.

When $z = f(x,y)$ and we fix z-values (heights), we call these traces **level curves**.

A collection of level curves is called a **contour map** (or **elevation map**).

Contour Map (Elevation Map) of Mt. St. Helens from 1979:

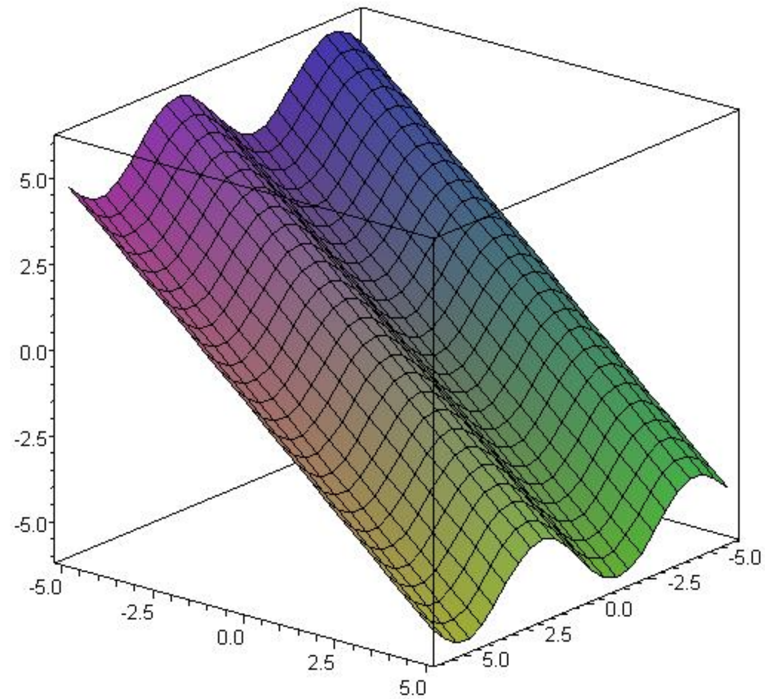


Example: Draw a contour map for

$$z = f(x, y) = y - x$$

Example: Draw a contour map for

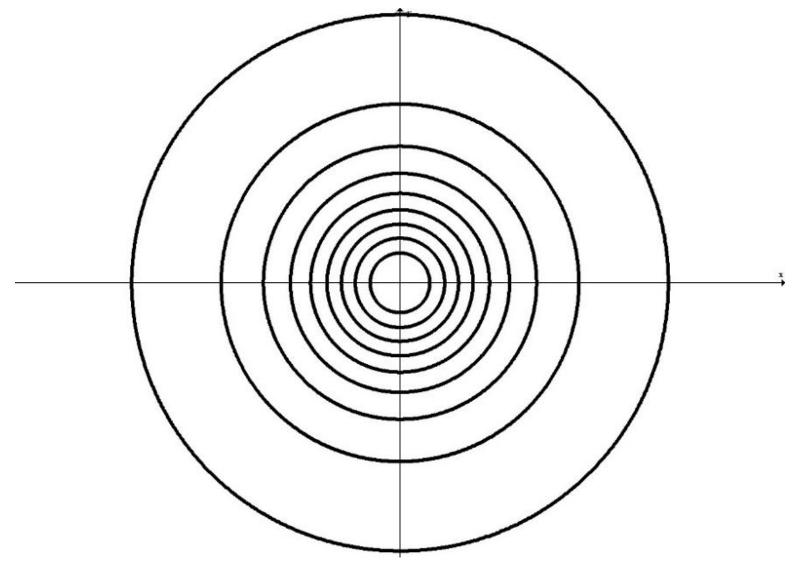
$$z = \sin(x) - y$$

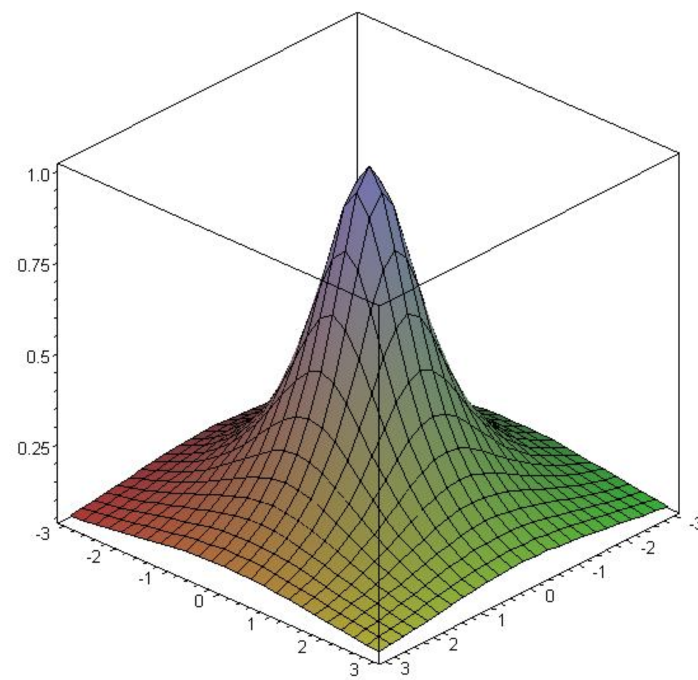


Example: Draw a contour map for

$$z = f(x, y) = \frac{1}{1 + x^2 + y^2}$$

(use z = 1/10, 2/10, … , 9/10, 10/10)



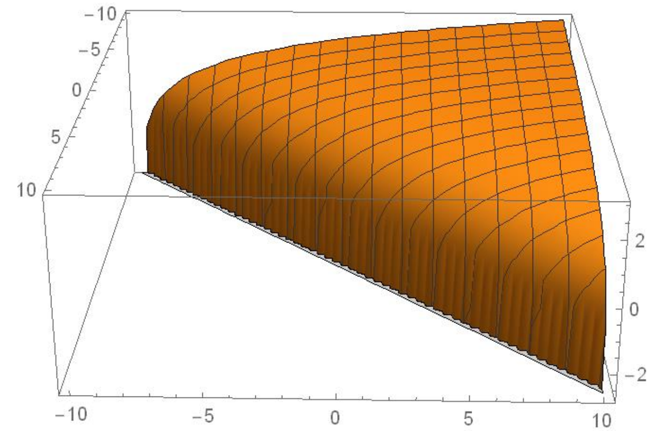


A question that asks "find the ***domain***" is asking if you know your functions well enough to understand when they are defined and not defined.

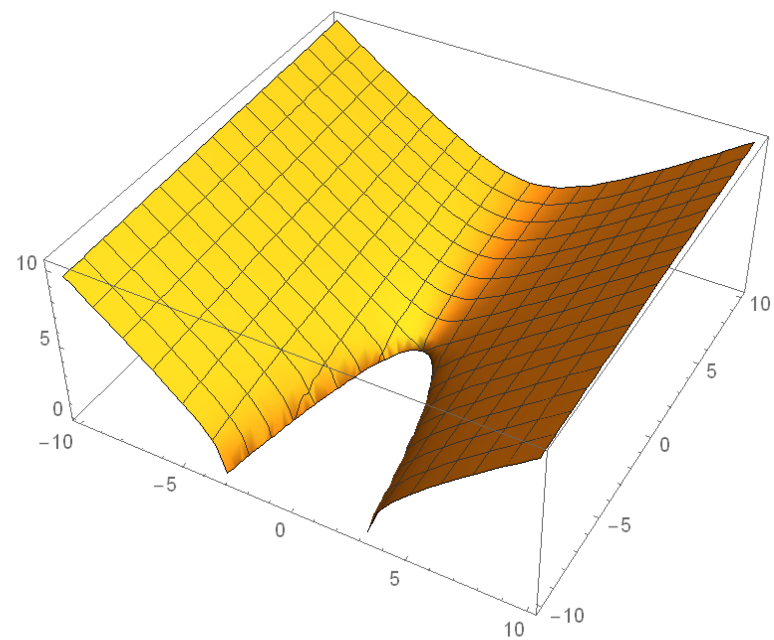
Appears in Function	*Restriction*
$\sqrt{BLAH}$	BLAH ≥ 0
STUFF/BLAH	BLAH ≠ 0
ln(BLAH)	BLAH > 0
$\sin^{-1}$(BLAH)	-1 ≤ BLAH ≤ 1
and other trig…	

Examples: Sketch the domain of

(1) $f(x,y) = \ln(y - x)$



(2) $g(x,y) = \sqrt{y + x^2}$



14.3 Partial Derivatives

Goal: Get the slope in two different directions on a surface.

Recall the key def'n for all calculus

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Today we define:

$$\frac{\partial z}{\partial x} = f_x(x, y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial z}{\partial y} = f_y(x, y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

Motivation: Consider

$$f(x,y) = x^2y + 5x^3 + y^2$$

Find

a. $\frac{d}{dx}[f(x,2)] = \frac{d}{dx}[x^2(2) + 5x^3 + (2)^2]$

b. $\frac{d}{dx}[f(x,3)] = \frac{d}{dx}[x^2(3) + 5x^3 + (3)^2]$

c. $\frac{d}{dx}[f(x,c)] = \frac{d}{dx}[x^2(c) + 5x^3 + (c)^2]$

Example:

$$f(x, y) = x^3 y + x^5 e^{xy^2} + \ln(y)$$

Example:

$$g(x, y) = \cos(x^3 + y^4)$$

Important Note on Variables

A variable can be treated as:

1. A constant
2. An independent variable (input)
3. A dependent variable (output),

Examples:

a) **One variable function of *x***:

$$y = x^2$$

$$\frac{dy}{dx} =$$

b) **Related rates**:

At time *t* assume a particle is moving along the path $y = x^2$.

$$\frac{dy}{dt} =$$

c) **Implicit functions**: $x^2 + y^2 = 1$

$$\frac{dy}{dx} =$$

d) **Multivariable**:

$$z = x^2 + y^3e^{6y} - 5xy^4$$

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

e) **Multivariable Implicit**:

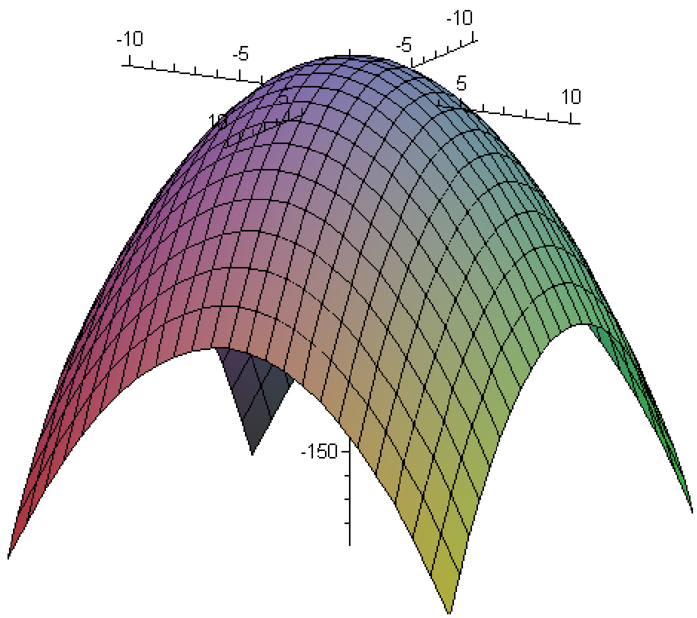
$$x^2 + y^2 - z^2 = 1$$

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

Graphical Interpretation:
Pretend you are skiing on the surface

$$z = f(x, y) = 15 - x^2 - y^2.$$



Exercises:

1. Find $f_x(x, y)$ and $f_y(x, y)$

2. Assume you are standing on the point on the surface corresponding to (x,y) = (4,7).

   Compute:

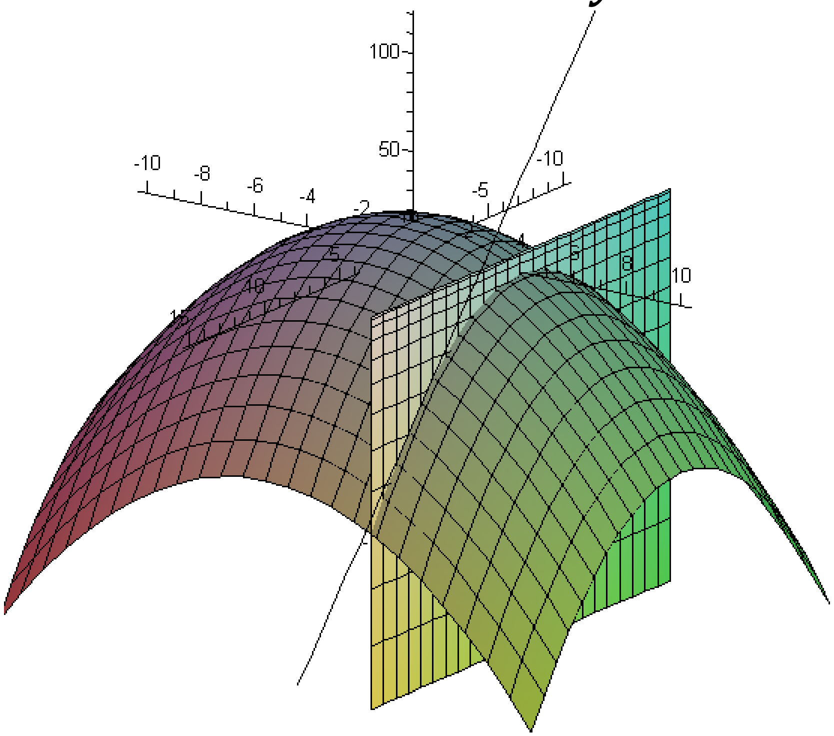
   i) $f(4,7) =$

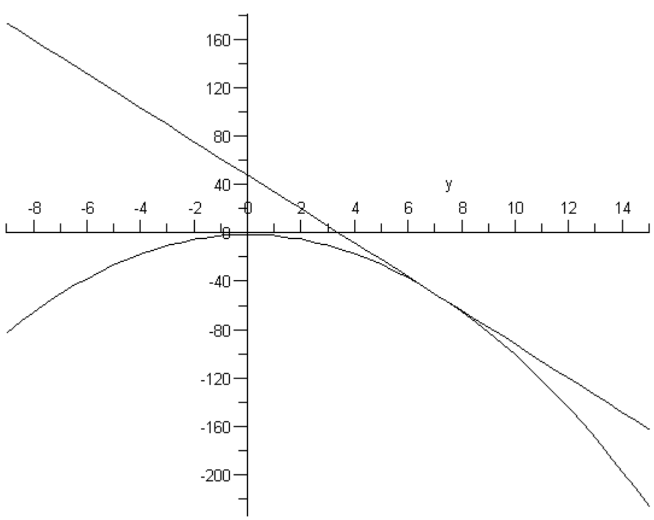
   ii) $f_x(4,7) =$

   iii) $f_y(4,7) =$

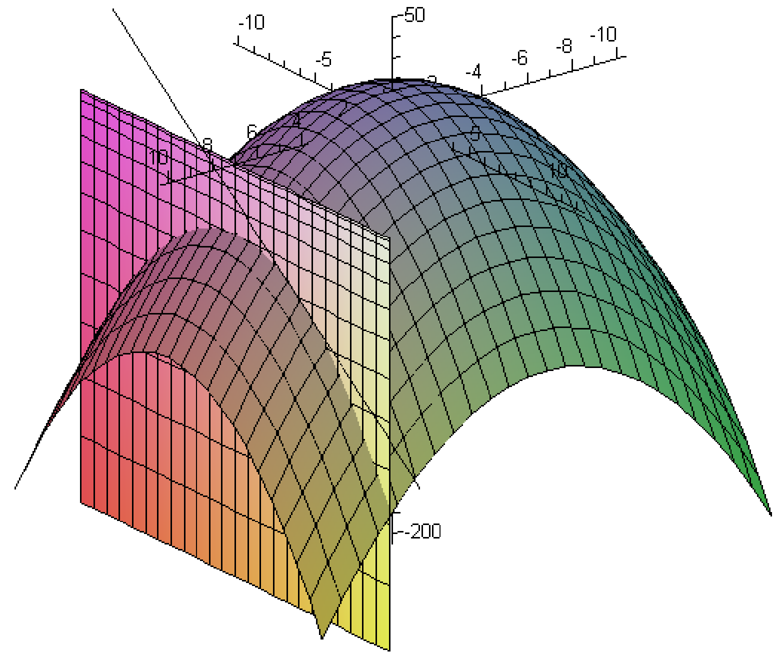
   What do these three numbers represent?

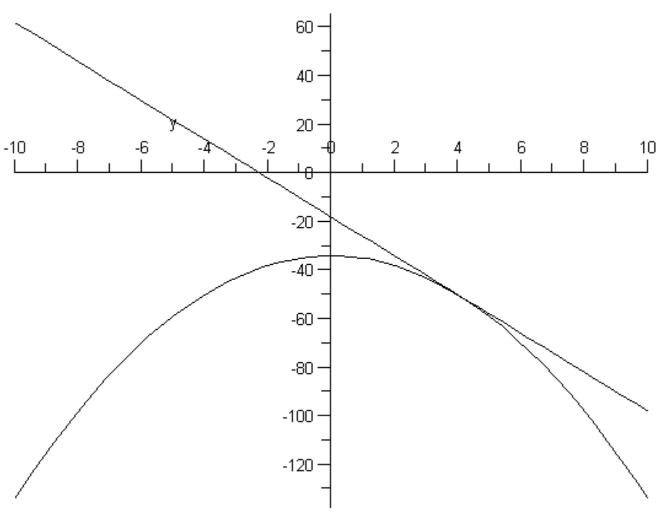
The plane y = 4 intersecting the surface $z = 15 - x^2 - y^2$.





The plane x = 7 intersecting the surface $z = 15 - x^2 - y^2$.





Second Partial Derivatives

Concavity in *x*-direction:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = f_{xx}(x, y)$$

Concavity in *y*-direction:

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = f_{yy}(x, y)$$

Mixed Partials:

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = f_{xy}(x, y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = f_{yx}(x, y)$$

Example: Find all second partials for
$z = f(x, y) = x^4 + 3x^2y^3 + y^5$